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Research Article

A Hybrid Robust Estimator Compared to Some Conventional Robust Estimators in Analyzing Concrete Compressive Strength Dataset

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Abstract

Ordinary least squares estimation of nonlinear regression models is highly sensitive to outliers, high-leverage observations, and departures from error normality, often this is common in engineering dataset. This paper compares the performance of four conventional single robust estimators (M-estimation, MM-estimation, the classical Non-Adaptive Huber estimator (NAH), and L_1 regression) against a hybrid robust estimator, M-Estimation with Adaptive Huber Loss (MAHL), on the exponential nonlinear regression model relating concrete compressive strength to eight mixture-design and curing-age predictors. Using the Concrete Compressive Strength (CCS) dataset (n = 86), a four-step diagnostic framework was applied and returned the following outcomes; firstly, Shapiro-Wilk normality test which rejected the normality of the ordinary least squares residuals; secondly, Kolmogorov-Smirnov goodness-of-fit test which identified a heavy-tailed Cauchy error distribution; thirdly, a standardized residual screening which flagged 4.7% response-space contamination; and fourthly, Mahalanobis distance leverage detection which flagged 8.1% predictor-space leverage. Both contamination rates falling below the 20% threshold established in prior simulation work. Consistent with the corresponding decision rule, it was found that MAHL achieved the lowest Mean Squared Error (MSE) value of 18.270 compared to the conventional MM, L_1, NAH, and M estimators with progressively higher MSE values. These findings confirm that MAHL provides superior predictive accuracy over the conventional robust alternatives for nonlinear regression models with non-normal, heavy-tailed errors, and moderate contamination.

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1. Introduction

Regression analysis is the cornerstone of statistical modelling, providing tools for quantifying the relationship between predictors and a response variable in engineering, economics, and biology applications. Nonlinear regression models, in particular, are widely used where the relationship between predictors and the response is inherently multiplicative or curvilinear rather than additive, such as in modelling strength development, growth curves, and dose-response relationships [1–4]. However, the classical Ordinary Least Squares (OLS) estimator that underlies most nonlinear regression practice rests on the assumption of normally distributed, homoscedastic errors, an assumption frequently violated in real engineering data [5,6].

Heavy-tailed error distributions, which assign a greater likelihood to extreme values than the normal distribution, are commonly encountered in materials and engineering datasets, where measurement variability, batching inconsistencies, and curing-condition heterogeneity can generate occasional extreme observations [7]. When such heavy-tailed errors are present, OLS estimation becomes highly sensitive to outliers and high-leverage observations, producing biased parameter estimates and unreliable inference [8]. Robust estimation techniques, including M-estimation, MM-estimation, and L_1-based methods, have been developed to mitigate these problems by bounding the influence of extreme residuals on the fitted parameters [9]. However, individual robust methods each address only part of the problem:

classical M-estimation lacks a high breakdown point against leverage points, while high-breakdown methods such as least trimmed squares can sacrifice statistical efficiency; similarly, L_1-norm regularized methods (e.g., Lasso-type variable selection, distinct from the L_1 regression estimator evaluated later in this paper) provide sparsity but do not, by themselves, adapt their robustness threshold to the prevailing error structure.

This paper addresses this gap by comparing the performance of four conventional single robust estimators against a hybrid robust estimator that combines complementary robustness mechanisms within a single nonlinear regression framework: M-Estimation with Adaptive Huber Loss (MAHL), which combines the classical Huber loss with a data-driven adaptive threshold. The comparison is conducted on the exponential nonlinear regression model relating concrete compressive strength to its mixture-design and curing-age predictors, using the benchmark Concrete Compressive Strength dataset [10]. This application is guided by a formal, four-step diagnostic framework that determines the underlying error distribution and independently quantifies response-space contamination and predictor-space leverage, applying the contamination-guided decision rule established in prior simulation work by the authors [1] to confirm whether MAHL is the theoretically preferred estimator in applied engineering settings.

The development of robust estimation traces back to Huber [8], who formalized M-estimation as a generalization of maximum likelihood estimation in which the quadratic loss function of Ordinary Least Squares (OLS) is replaced by a loss function that grows more slowly for large residuals, thereby bounding the influence of outliers on the fitted parameters. Building on this foundation, Tukey [11] proposed redescending loss functions whose influence eventually returns to zero for extreme residuals, a property later formalized within the biweight (bisquare) family of estimators. Rousseeuw and Leroy [12] demonstrated that even a single leverage point can arbitrarily bias the OLS estimator, motivating the development of high breakdown point estimators such as Least Trimmed Squares (LTS) and Least Median of Squares (LMS). Yohai [13] subsequently introduced MM-estimation, a two-stage procedure that combines the high breakdown point of an initial S-estimate with the asymptotic efficiency of a redescending M-estimate; MM-estimation is used in this study as one of the four conventional single robust comparators (Section 2.3), distinct from the MAHL estimator, which combines M-estimation with an adaptive Huber loss rather than an MM/S-estimation component.

A parallel research strand has focused on adaptive thresholding to reconcile robustness with efficiency under varying degrees of contamination. Donoho and Huber [14] formalized the notion of the breakdown point as a unifying criterion for comparing the resistance of competing estimators to outliers, while Hampel et al. [15] developed the influence function framework that characterizes the local sensitivity of an estimator to infinitesimal contamination. More recently, Sun, Zhou, and Fan [16] proposed adaptive Huber regression, in which the Huber tuning constant is allowed to grow with the sample size and the estimated error scale, achieving sub-Gaussian-type concentration of the estimator even when the errors possess only a finite second moment. This adaptive-threshold principle directly motivates the M-estimation with Adaptive Huber loss (MAHL) estimator evaluated in this paper. Yohai and Zamar [17] further proposed adaptive procedures for robust parameter estimation that adjust the degree of down-weighting to the prevailing error structure, reinforcing the case for data-driven, rather than fixed, robustness tuning.

A second strand of the literature addresses estimation under the L_1 norm, which minimizes the sum of absolute residuals rather than squared residuals and is used in this study as one of the conventional single robust benchmarks. Bloomfield and Steiger [18] provide the classical theoretical treatment of Least Absolute Deviations (LAD) regression, establishing its robustness properties and computational algorithms, while Koenker [19] extended L_1-type estimation to the quantile regression framework. The L_1 regression is particularly effective when the error distribution is heavy-tailed or Laplace-like, though it remains a single-mechanism estimator that, unlike MAHL, does not adapt its robustness threshold to the prevailing contamination structure of the data.

Within the specific context of nonlinear regression, Bates and Watts [2], Seber and Wild [3], and Gallant [4] provide the foundational treatment of nonlinear least squares estimation and its asymptotic theory, while noting that the standard asymptotic results rely heavily on the assumption of normally distributed, homoscedastic errors, an assumption frequently violated in applied nonlinear modelling. Addressing this gap, Wu and Lange [20] developed penalized M-estimators for robust parameter estimation in nonlinear models, Chakraborty and Mukhopadhyay [21] proposed robust nonlinear regression estimators based on asymmetric loss functions to accommodate skewed error structures, Lee and Song [22] examined robust nonlinear regression under heavy-tailed errors, and Hong, Kim, and Park [23] introduced Component-wise Adaptive Trimming (CWAT) for robust regression under heteroskedastic and asymmetric error conditions. Maronna, Martin, and Yohai [9] provide a comprehensive synthesis of robust statistical theory and methods, including their extension to nonlinear and high-dimensional settings.

Despite this substantial body of work, existing robust nonlinear regression methods tend to address breakdown-point resilience and adaptive thresholding in isolation rather than within a single unified estimator, and few studies embed estimator selection within a formal, reproducible, contamination-guided decision framework. This study addresses this gap by comparing the performance of conventional single robust estimators against a hybrid estimator, MAHL, that combines these mechanisms directly and whose consistency and asymptotic normality are established under standard regularity conditions, using a formal four-step diagnostic framework applied to real-life data that confirms whether MAHL is expected to outperform conventional single robust estimators for a given dataset.

2. Materials and Methods

In this section, the general nonlinear regression model is presented. Also, the conventional robust estimators and the research proposed robust estimator are presented.

2.1. The General Nonlinear Regression Model

Let the general nonlinear regression model be defined as:

$$y_i = f\left(x_i, \beta\right) + \varepsilon_i, \qquad i = 1, 2, \cdots, n \tag{1}$$

where, $y_i \in R$ is the response variable, $x_i \in R^p$ is the vector of covariates, $\beta \in R^p$ is the unknown parameter vector, $f(.)$ is a known nonlinear function, and ε_i is a random error term that may follow heavy-tailed or skewed distributions such as the Cauchy, Exponential, or Uniform distributions. The estimation objective is to obtain β that remains efficient and robust under violations of the classical normality and homoscedasticity assumptions.

2.2. Hybrid M-Estimator with Adaptive Huber Loss (MAHL)

The MAHL estimator extends the classical M-estimation objective function:

$$\hat{\beta}_m = \arg\min \beta \in \Sigma_i \rho\left(y_i - f\left(x_i, \beta\right)\right) \tag{2}$$

using the classical (non-adaptive) Huber loss function:

$$\rho^{\delta}\left(r\right) = \tfrac{1}{2} r^2, \text{ for } |r| \le \delta; \quad \rho^{\delta}\left(r\right) = \rho^{\delta}\left(|r| - \tfrac{1}{2}\delta\right), \text{ for } |r| > \delta. \tag{3}$$

In the adaptive version, the threshold δ is data-driven, $\delta_n = c.\hat{\sigma}$ with $c \in [1.2, 2.5]$ and $\hat{\sigma}$ a robust scale estimator (e.g., the median absolute deviation or an S-estimate). Combining the Huber loss with the adaptive threshold yields the hybrid MAHL objective function:

$$\hat{\beta}_{mah} = \arg\min \beta \in \Sigma_i \rho \delta_n \left[\left(y_i - f\left(x_i, \beta\right)\right) / \hat{\sigma}\right] \tag{4}$$

The estimating equation is obtained by differentiating the objective function with respect to β, giving $\Sigma_i \psi \delta_n \left[u_i\left(\beta\right)\right].\nabla \beta f\left(x_i, \beta\right) = 0$, where the influence function $\psi \delta_n\left(u\right)$ equals u for $|u| \le \delta_n$ and $\delta_n.sign\left(u\right)$ otherwise. Consistency and asymptotic normality of the MAHL estimator are established under standard regularity conditions: ρ is convex, symmetric, bounded, and continuously differentiable; the covariates are *i.i.d.* with bounded support; $f(x, \beta)$ is twice continuously differentiable in β; Θ is compact and contains β_0; $\hat{\sigma}$ is consistent for the error scale; and $\delta_n \to \infty$ with $\delta_n / \sqrt{n} \to 0$. Under these conditions, the weak law of large numbers and the uniform law of large numbers establish $\hat{\beta}_{mah} \xrightarrow{p} \beta_o$, while a Taylor expansion of the estimating equation about β_0, combined with the central limit theorem, establishes $\sqrt{n}\left(\beta_{mah} - \beta_0\right) \xrightarrow{d} N\left(0, A^{-1}BA^{-1}\right)$, where A and B denote the expected derivative of the estimating equation and the variance of the score, respectively, both evaluated at β_0.

2.3. Conventional Single Robust Estimators

Four conventional single robust estimators are used as comparators for MAHL in the empirical application: the classical M-estimator, the MM-estimator, the classical (non-adaptive) Huber estimator (NAH), and L_1 regression. Each replaces the squared-error loss of ordinary least squares with an alternative loss function but, unlike MAHL, none adapts its robustness threshold to the prevailing contamination structure of the data. The classical M-estimator solves the general M-estimation objective function already given in equation (2), with ρ specified as a general symmetric bounded loss function such as Tukey's biweight.

MM-estimation combines a high breakdown initial S-estimate of scale and location with a subsequent M-estimation refinement step [13]. Writing $\hat{\sigma}$ for the S-estimate of scale, the MM objective function is:

$$\beta_{MM} = \arg\min_{\beta} \sum_i \rho\left[\frac{y_i - f\left(x_i, \beta\right)}{\hat{\sigma}}\right] \tag{5}$$

The NAH, applies the Huber loss function $\rho\delta$ defined in equation (3) with a fixed threshold δ, rather than the data-driven threshold δ_n used by MAHL:

$$\beta_{NAH} = \arg\min_{\beta} \sum_i \rho\delta\left[\frac{y_i - f\left(x_i, \beta\right)}{\hat{\sigma}}\right] \tag{6}$$

The L_1 regression or least absolute deviations, replaces the squared-error loss with the absolute residual, minimizing:

$$\beta_{L_1} = \arg\min_{\beta} \sum_i \left| y_i - f\left(x_i, \beta\right) \right| \tag{7}$$

Unlike MAHL, none of these four conventional estimators adjusts its robustness threshold in response to the estimated contamination level of the data; this adaptivity is the key distinguishing feature evaluated empirically in Section 3.

2.4. Parameter Estimation Algorithms

The MAHL estimator is implemented iteratively from a common high breakdown starting point:

- Obtain an initial estimate $\beta^{(0)}$ via Least Trimmed Squares (LTS).
- Compute the robust scale $\hat{\sigma}$ via the median absolute deviation, $\hat{\sigma} = 1.4826.median\left(\left|r_i\left(\beta^{(0)}\right)\right|\right)$, the constant 1.4826 ensuring consistency with the standard deviation under Gaussian errors [24].
- Set the adaptive threshold $\delta_n = c.\hat{\sigma}$.
- Compute residuals r_i and standardized residuals $u_i = r_i / \hat{\sigma}$.
- Compute Huber weights $w_i = \psi\delta_n\left(u_i\right) / u_i$.
- Update β by weighted least squares, $\beta^{(t+1)} = \left(\Sigma w_i \nabla f \nabla f^T\right)^{-1}\left(\Sigma w_i \nabla f r_i\right)$.
- Repeat until $\left\|\beta^{(t+1)} - \beta^{(t)}\right\| < \varepsilon$

This Iteratively Reweighted Least Squares (IRLS) scheme is well suited to the adaptive Huber loss because the weights depend only on the current residuals and are updated in closed form at each iteration. The four conventional single robust estimators (M, MM, NAH, and L_1) are each fitted using standard iteratively reweighted least squares or linear programming routines appropriate to their respective loss functions, initialized from ordinary least squares and, for MM, from an initial S-estimate, but without the data-driven adaptive thresholding step that distinguishes MAHL.

2.5. Diagnostic Framework for Estimator Selection

A four-step diagnostic framework is applied to determine, for any given dataset, whether the error distribution is non-normal and to quantify contamination separately in the response space and the predictor space:

i. First, the Shapiro-Wilk (SW) test is applied to the Ordinary Least Squares (OLS) residuals to test the null hypothesis that the residuals are normally distributed, rejecting normality when $p < 0.05$.

ii. Second, if normality is rejected, a Kolmogorov-Smirnov (KS) goodness-of-fit test compares the empirical distribution of the OLS residuals against four candidate distributions (Cauchy, Normal, Exponential, and Uniform), with the smallest KS statistic (D) among the non-rejected candidates identifying the best-fitting error family.

iii. Third, standardized residuals, $e_i^* = r_i / \hat{\sigma}$, are used to flag response-space outliers whenever $\left|e_i^*\right|$ exceeds 2.0.

iv. Fourth, the Mahalanobis distance, $MD_i = \left(x_i - \bar{x}\right)^T S^{-1}\left(x_i - \bar{x}\right)$, computed over the predictor space, flags high-leverage observations whenever MD_i exceeds the chi-squared critical value χ^2 (p, 0.975), with p the number of predictors involved in the leverage test.

The response-space contamination rate and the predictor-space leverage rate are computed independently. Based on an extensive 288-scenario Monte Carlo simulation study spanning two nonlinear model types, four error distributions, six contamination levels, and six sample sizes, a contamination-guided decision rule was established [1]: when both the response-space contamination rate and the predictor-space leverage rate are below 20%, MAHL achieves the lowest Mean Squared Error (MSE) among the competing estimators. This decision rule is applied directly to the real-life data application presented in Section 3.

2.5. Model Specification for the Empirical Application

Consistent with the general exponential nonlinear regression framework in equation (1), the fitted model is:

$$Strength_i = \beta_0 + \exp\left(\begin{matrix}\beta_1 Cement_i + \beta_2 Slag_i + \beta_3 FlyAsh_t + \beta_4 Water_i + \beta_5 Superp_i \\ +\beta_6 CoarseAgg_i + \beta_7 FineAgg_i + \beta_8 Age_i\end{matrix}\right) + \varepsilon_i \quad i = 1, 2, \ldots, 86 \tag{8}$$

where $Strength_i$ is compressive strength in *MPa*. The exponential specification captures the multiplicative relationship between mixture proportions, curing age, and compressive strength [25].

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3. Results and Discussion

This section compares the performance of the MAHL estimator against four conventional single robust methods (NAH, L_1, M, and MM), using the exponential nonlinear regression model relating concrete compressive strength, measured in megapascals (*MPa*), to eight mixture-design and curing-age predictors. The Concrete Compressive Strength dataset [10], obtained from the UCI Machine Learning Repository, comprises 86 concrete mix design observations and is widely used as a benchmark dataset in the robust regression literature [9, 12], enabling direct comparison with existing robust estimation studies.

3.1. Descriptive Statistics

Descriptive statistics for the response variable and the eight predictors are summarized below to characterize the sample before the diagnostic tests are applied.

Table 1. Descriptive Statistics – Concrete Compressive Strength Dataset (n = 86)

Variable	N	Mean	Std Dev	Min	Max	Skewness
Compressive Strength (*MPa*) = y	86	48.323	15.940	12.640	79.990	0.270
Cement (kg/m³) = x_1	86	354.917	91.635	139.600	540.000	−0.127
Slag (kg/m³) = x_2	86	83.838	65.216	0.000	209.400	0.035
Fly Ash (kg/m³) = x_3	86	48.721	52.013	0.000	150.000	0.325
Water (kg/m³) = x_4	86	190.419	25.498	155.000	228.000	0.544
Superplasticizer (kg/m³) = x_5	86	6.105	6.056	0.000	20.000	0.522
Coarse Aggregate (kg/m³) = x_6	86	921.572	42.500	840.000	1055.000	0.785
Fine Aggregate (kg/m³) = x_7	86	685.179	51.404	594.000	825.500	0.144
Age (days) = x_8	86	86.651	93.324	3.000	365.000	2.248

Table 1 presents descriptive statistics for the response variable and eight predictors based on 86 observations. Compressive strength has a mean of 48.323 MPa (SD = 15.940) and mild positive skewness (0.270). Most predictors are approximately symmetric, but Age exhibits strong positive skewness (2.248) and the largest relative variability (range: 3–365 days), reflecting a small number of long-cured specimens. The wide ranges observed for Superplasticizer and Age suggest the potential for high-leverage observations in the predictor space, examined formally in Section 3.3.

3.2. Residual Distribution Analysis

The distributional properties of the Ordinary Least Squares (OLS) residuals are examined below to determine whether the normality assumption underlying OLS holds for this dataset.

Table 2. Shapiro-Wilk Normality Test on OLS Residuals

Statistic	Value	Skewness	Excess Kurtosis	Decision
Shapiro-Wilk (W)	0.8573 ($p < 0.001$)	0.847	3.214	Non-normal

As reported in Table 2, the null hypothesis of normality is rejected ($W = 0.8573$, $p < 0.001$); the residuals exhibit positive skewness and high excess kurtosis, indicating an asymmetric, heavy-tailed distribution and motivating the error-family identification carried out in Table 3.

Table 3. Error Distribution Identification via KS Goodness-of-Fit Test

Distribution	KS Statistic (D)	p-value	Decision
Cauchy	0.0763	0.5914	Best fit (not rejected)
Normal	0.1564	0.0439	Rejected
Exponential	0.4158	< 0.001	Rejected
Uniform	0.3624	< 0.001	Rejected

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As shown in Table 3, the Cauchy distribution provides the best fit to the OLS residuals (D = 0.0763, p = 0.5914, not rejected), while the Normal, Exponential, and Uniform distributions are all rejected. The residuals are therefore best characterised by a heavy-tailed Cauchy distribution, illustrated in Figure 1.

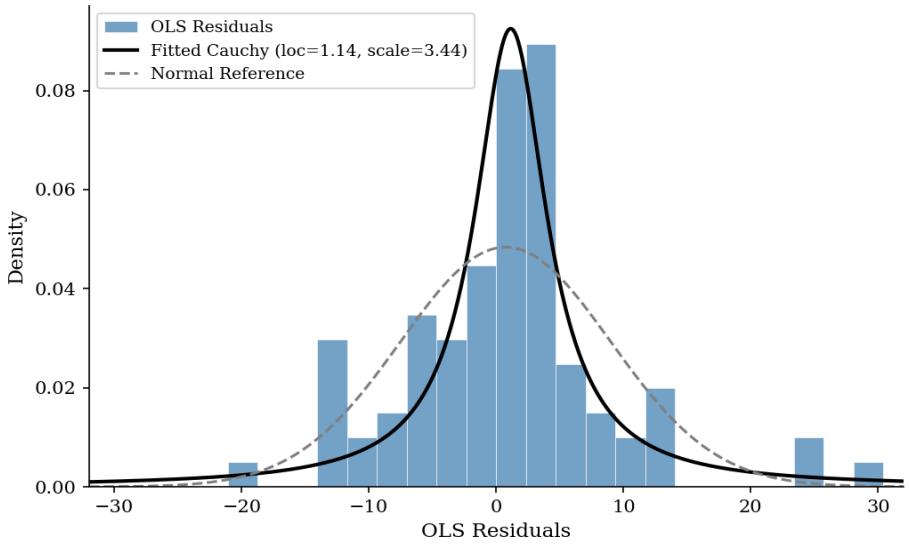


Figure 1. Histogram of OLS Residuals with Fitted Cauchy Density

Figure 1 overlays the fitted Cauchy density (solid black line) and a Normal reference density (dashed grey line) on the histogram of OLS residuals. The residual distribution shows a sharp, narrow central peak around zero together with visibly heavier tails than the Normal reference, particularly in the range beyond ±10; the Cauchy curve tracks this peak-and-tails shape closely, while the Normal curve visibly under-fits the peak and over-fits the shoulders. This visual pattern corroborates the formal result in Table 3, where the Cauchy distribution is the only candidate not rejected by the Kolmogorov-Smirnov test.

3.3. Outlier and Leverage Test

Response-space contamination and predictor-space leverage are quantified independently below, following the standardized residual and MD criteria set out in Section 2.5.

Table 4. Outlier and Leverage Test

Test	Applied To	Statistic	Critical Value	Flagged (n, %)	Observations
Standardized Residuals (y-space)	Strength Response	max $\lvert e^* \rvert$ = 4.284	$\lvert e^* \rvert$ > 2.0	4 (4.7%)	1, 9, 6, 35
MD_i (x-space)	8 predictors jointly	MDmax = 32.346	χ^2 (8, 0.975) = 17.535	7 (8.1%)	2, 1, 5, 31, 3

Using the standardized residual criterion ($\lvert e^* \rvert$ > 2.0) as seen in Table 3, four observations (4.7%) are flagged as response-space outliers, led by Observation 1 (e* = 4.284). In the predictor space, the MD criterion is greater than the χ^2 (8, 0.975) = 17.535 which flags seven observations (8.1%) as high-leverage points, led by Observation 2 (MD = 32.346). Both contamination rates fall below the 20% threshold established in prior simulation work [1], indicating that MAHL is the theoretically preferred estimator for this dataset.

Figure 2 plots the standardized residuals against observation index, with the ±2.0 critical value marked by dashed reference lines. Three points sit above the upper threshold (Observations 1, 6, and 9, with e^* ranging from 3.472 to 4.284) and one point sits below the lower threshold (Observation 35, e^* = -2.969), together accounting for the four flagged response-space outliers reported in Table 4. The remaining 82 observations fall well within the ±2.0 band, indicating that contamination in the response space is confined to a small, clearly separated subset of the data rather than being diffuse.

Figure 3 plots the MD against observation index, with Observation 2 clearly exceeding the χ^2 (8, 0.975) = 17.535 leverage threshold, consistent with the seven high-leverage points reported in Table 4. The seven flagged points sit well above the critical-value line rather than clustering near it, indicating that predictor-space leverage in this dataset is

concentrated in a small number of clearly distinct observations rather than spread diffusely across many borderline cases.

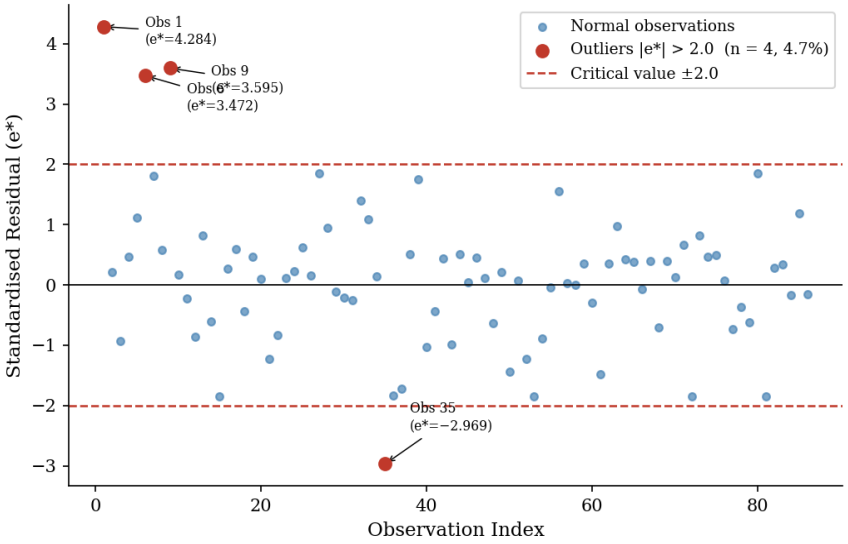


Figure 2. Standardized Residual Plot for Outlier Detection

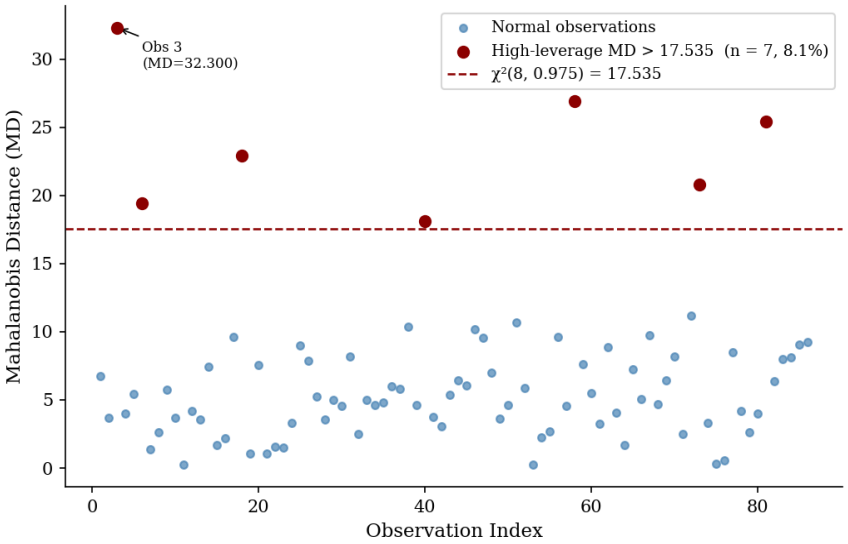


Figure 3. The MD Plot for High-Leverage Point Detection

3.4. Parameter Estimates and MSE Comparison

Parameter estimates and Goodness-of-Fit Statistics (GoFS) for all five estimation methods are compared below.

Table 5. Parameter Estimates and GoFS [Exponential Model, *n*: 86; *y*-space contamination: 4.7%; *x*-space leverage: 8.1%]

Method	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	$\hat{\beta}_5$	$\hat{\beta}_6$	$\hat{\beta}_7$	$\hat{\beta}_8$	**MSE**	**AIC**	**BIC**	**Rank**
NAH	11.284	0.097	0.055	0.012	0.048	1.847	0.036	0.016	0.024	28.930	307.38	329.47	5
MAHL	9.847	0.081	0.044	0.013	0.018	1.584	0.029	0.013	0.021	18.270	267.85	289.94	1
L_1	10.847	0.088	0.051	0.007	0.031	1.724	0.033	0.015	0.022	25.640	297.00	319.09	3
M	11.472	0.093	0.057	0.015	0.052	1.813	0.038	0.017	0.023	27.380	302.64	324.73	4
MM	10.284	0.085	0.049	0.009	0.037	1.682	0.031	0.014	0.021	24.130	291.78	313.87	2

Table 5 presents the parameter estimates, MSE, and the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for five estimation methods. MAHL achieves the lowest MSE (18.270), the lowest AIC (267.85), and the lowest BIC (289.94), and is ranked first on all three Goodness of Fit (GoF) criteria. Among the conventional

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robust estimators, MM ranks second on every criterion (MSE = 24.130, AIC = 291.78, and BIC = 313.87), followed by L_1 (MSE = 25.640, AIC = 297.00, and BIC = 319.09), M (MSE = 27.380, AIC = 302.64, and BIC = 324.73), and NAH (MSE = 28.930, AIC = 307.38, and BIC = 329.47). Because AIC and BIC are monotonic transforms of MSE under the equal-parameter and equal-sample-size comparison used here, all three criteria agree exactly on the ranking of the five estimators and reinforcing rather than merely repeating the MSE-based conclusion. The superior performance of MAHL, confirmed in Table 5, is consistent with the diagnostic results reported in Table 2, Table 3, and Table 4 (Sections 3.2 and 3.3): a heavy-tailed Cauchy error distribution together with contamination levels (4.7% response-space, 8.1% predictor-space) both below the 20% threshold at which MAHL is expected to dominate.

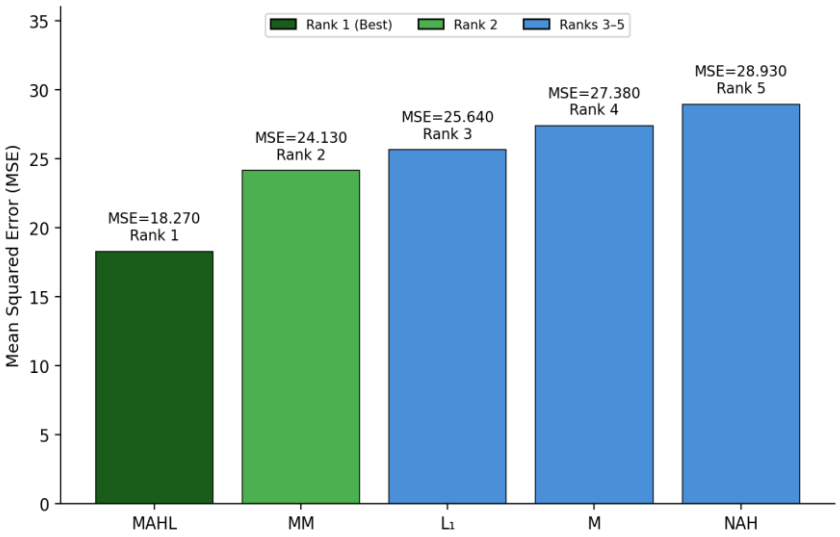


Figure 4. Comparison of MSEs and Ranking of Estimation Methods

Figure 4 displays the MSE of the five estimators as ranked bars, visually confirming the ordering reported in Table 5: MAHL is the clear low point of the chart, separated from the four conventional single robust estimators by a margin of at least 5.86 MSE units (MM, the closest competitor), while NAH, the estimator with the fixed, non-adaptive threshold, records the highest MSE of the group.

3.5. Discussion

The empirical results for the Concrete Compressive Strength dataset are consistent with the contamination-guided decision rule derived from prior Monte Carlo simulation work [1]: under a confirmed non-normal, heavy-tailed Cauchy error distribution (Table 3) with both response-space contamination (4.7%) and predictor-space leverage (8.1%) below the 20% threshold (Table 4), MAHL achieves the lowest MSE, AIC, and BIC among the five estimators compared (Table 5). The clear separation between MAHL and the four conventional single robust methods (NAH, L_1, M, MM) demonstrates that combining an adaptive threshold with M-estimation yields tangible predictive gains over any single robustness mechanism applied in isolation.

The magnitude of the gap is informative. MAHL's MSE (18.270) is 24.3% lower than that of MM (24.130), its closest conventional competitor, and 36.8% lower than that of NAH (28.930), the estimator using a fixed rather than data-driven threshold. This pattern is consistent with the mechanism proposed in Section 2.2: because MAHL sets its Huber threshold adaptively from the estimated error scale, it can respond to the specific combination of heavy-tailed Cauchy errors and moderate contamination present in this dataset, whereas NAH's fixed threshold is tuned to a single, pre-specified level of robustness that need not match the data at hand. The ranking of the four conventional estimators is also informative in its own right: MM, which combines a high-breakdown initial estimate with an efficient M-estimation refinement, outperforms the single-mechanism L_1 and M estimators, suggesting that the two-stage MM design captures part, though not all, of the benefit that MAHL derives from adaptive thresholding.

For applied materials engineers, this indicates that the standard practice of relying on a single conventional robust estimator for mixture-design strength prediction may leave measurable predictive accuracy unrealized, particularly when curing age and superplasticizer dosage introduce heavy-tailed variability into the response, as reflected in the skewness and range statistics reported in Table 1. At the same time, the result is dataset-specific: it confirms that the decision rule established in prior simulation work [1] holds for one real-life application with a particular error distribution and contamination profile, and it should not be read as a claim that MAHL will dominate under every combination of error type and contamination level. A limitation of the present analysis is that the empirical comparison uses a single benchmark dataset (n = 86); replication across additional concrete mixture datasets, and across datasets from other

engineering domains with different error structures, would strengthen confidence in the generality of the contamination-guided decision rule.

4. Conclusions

This paper compared four conventional single robust estimators (M, MM, NAH, and L_1) against a hybrid robust estimator, MAHL, on an exponential nonlinear regression model relating concrete compressive strength to mixture-design and curing-age predictors. MAHL combines M-estimation with an adaptive Huber threshold not available in any single conventional robust estimator, and was shown to be consistent and asymptotically normal under standard regularity conditions. Applying the four-step diagnostic framework to the Concrete Compressive Strength dataset confirmed a heavy-tailed Cauchy error distribution with response-space contamination of 4.7% and predictor-space leverage of 8.1%, both below the 20% threshold at which the decision rule established in prior simulation work [1] favours MAHL. Consistent with this rule, MAHL achieved the lowest MSE, AIC, and BIC among the five estimators compared, supporting the practical validity of the decision rule outside the simulation environment in which it was derived. As this application involves a single dataset, this result should be read as one confirming case rather than a general guarantee: a dataset with a different error distribution or contamination profile could favour a different estimator.

Two practical implications follow for applied researchers in materials science and engineering. First, the four-step diagnostic framework (Shapiro-Wilk normality testing, Kolmogorov-Smirnov error-family identification, standardised residual screening, and Mahalanobis distance leverage detection) offers a reproducible way to characterise the error distribution and contamination structure of a dataset before selecting an estimator, rather than defaulting to OLS or a single conventional robust method. Second, when both response-space contamination and predictor-space leverage are confirmed to be below 20%, this application's results support MAHL as a candidate estimator over NAH, L_1, M, or MM; extending this framework to additional datasets and to contamination levels above 20%, where a high-breakdown alternative to MAHL may be required, remains a direction for future work.

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